

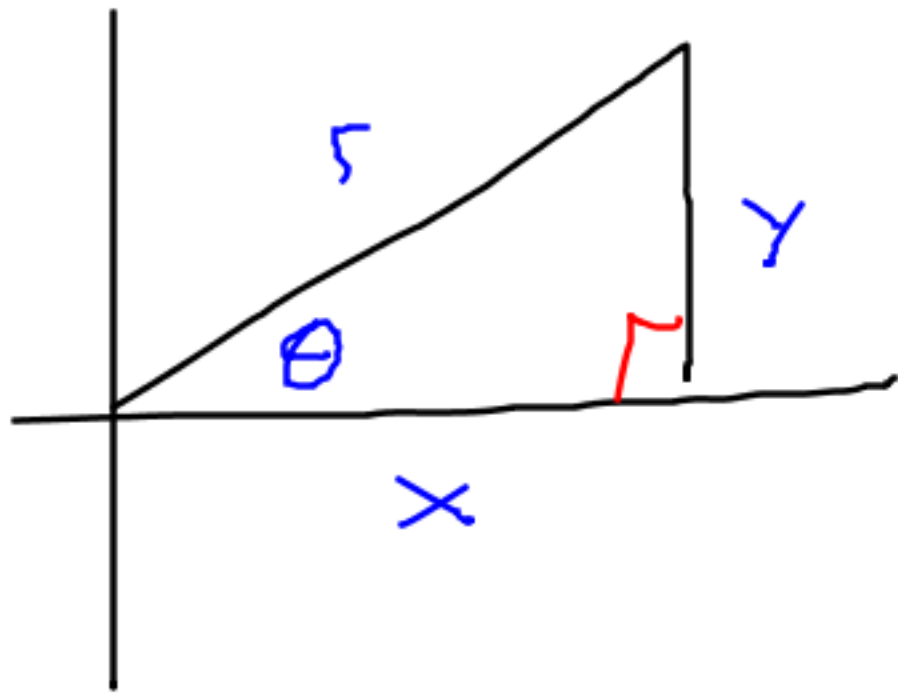
5.5 Trigonometric Identities

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5.5] Trigonometric Identities

$$\sin \theta + \cos \theta \cot \theta = \csc \theta$$

First recall:



$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

Our first identity:

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

L.S.

$$\frac{y}{r}$$

$$\frac{y}{r}$$

$$\frac{y}{r} \cdot \frac{r}{x}$$

$$\frac{y}{x} = \tan \theta$$

← memorize!

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\therefore \text{L.S.} = \text{R.S.}$$

Or second identity:

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \checkmark \checkmark$$

L.S.

$$\left(\frac{y}{r} \right)^2 + \left(\frac{x}{r} \right)^2$$

$$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$$

$$= \frac{\cancel{y}^2 + \cancel{x}^2}{r^2} = \frac{r^2}{r^2} = 1$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sin^2 \theta = \underline{1 - \cos^2 \theta}$$

$$\sin^2 \theta = (1 - \cos \theta)(1 + \cos \theta)$$

$$\therefore \text{L.S.} = \text{R.S.}$$

Prove: $\cos\theta \tan\theta = \sin\theta$

$$\text{L.S. } \frac{\cancel{\cos\theta}}{1} \left(\frac{\sin\theta}{\cancel{\cos\theta}} \right)$$

$$= \sin\theta$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

$$1 = \sec^2 \theta - \tan^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

L.S.

$$1 + \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{1}{\cos^2 \theta} = \sec^2 \theta$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$$\sec^2 \theta + \csc^2 \theta = \sec^2 \theta \csc^2 \theta$$

L.S. $\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta}$$

$$= \frac{1}{\cos^2 \theta \sin^2 \theta} = \sec^2 \theta \csc^2 \theta \therefore \text{L.S.} = \text{R.S.}$$

$$\therefore \sin \theta + \cos \theta \cot \theta = \csc \theta$$

$$\text{L.S.} \quad \sin \theta + \cos \theta \left(\frac{\cos \theta}{\sin \theta} \right)$$

$$= \frac{\sin \theta + \cos^2 \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} = \frac{1}{\sin \theta} = \csc \theta$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$$\sin \theta \cos \theta \tan \theta = 1 - \cos^2 \theta$$